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# Electrooptical switching properties of uniform layer tilted surface stabilized ferroelectric liquid crystal devices 

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#### Abstract

We present calculations of the light intensity transmitted through a ferroelectric liquid crystal light valve with uniformly tilted layers during electrooptical switching. We discuss the switching characteristics of the light valve for different layer tilts and at different applied voltage steps. We also show that the rise and fall times of the light intensity are in general different.


## 1. Introduction

The physics and device aspects of surface stabilized ferroelectric liquid crystals (SSFLCs) have attracted much attention. Experimental results have been reported and interpreted based on the assumptions that in SSFLC cells the molecular directors are uniformly arranged and smectic layers are perpendicular to the substrates [1-4]. Recently, several papers have been published concerning the smectic layer tilting with respect to substrates [5], the electrooptic switching of these layer tilted ferroelectric liquid crystal light valves [6] and the asymmetric electrooptic switching of the SSFLC cells [7].

In this paper, we extend the calculation of the electrooptic switching properties from the case with layers perpendicular to the substrate of [3], to the case of layers uniformly tilted to the substrates. As before [3], we ignore the distortions in the director that would arise from surface and elastic torques, since the characteristic size of these distortions is small compared to the light wavelength $\lambda$ whenever the applied electric field $E$ is large compared to $K / P \lambda^{2}$, where $K$ is the Oseen-Frank elastic constant and $P$ is the spontaneous polarization. For a typical FLC and $\lambda=1 \mu \mathrm{~m}$ this field has the small value of $0.3 \mathrm{~V} / \mu \mathrm{m}$ [2]. We also point out that an optical response asymmetric in rise and fall times can result solely from the geometrical arrangement of crossed polarizers relative to molecular orientations. This is simply because the optical transmission is not a linear function of azimuthal angle $\varphi$ and this angle is not a linear function of time [3].

## 2. Switching properties of the layer tilted FLC cells

The arrangement of smectic layers in surface stabilized ferroelectric liquid crystal cells has been determined experimentally to be tilted with respect to substrates [5]. The electrooptic response of these cells may be very different from the ones with layers perpendicular to the cell substrates. In this paper, we consider the case where the smectic layers are uniformly tilted at an angle $\delta$ with respect to the substrates and where the molecular directors are uniformly arranged, as shown in figure 1. The coordinate system is illustrated in figure 2.


Figure 1. Geometry of ferroelectric liquid crystal light valve. Liquid crystal slab of thickness $d$ is between glass plates with electrodes on their inner surfaces. Smectic layers make an angle of $\delta$ with the normal of slab faces. Polarizer makes an angle $\Omega$ to the $z$ axis.


Figure 2. Coordinate systems for director motion. Smectic layers define the $x l$ plane which is making an angle $\delta$ with the $x y$ plane defined by the $x$ axis and the slab surface normal. In smectic $\mathrm{C}^{*}$ phase, molecular director $\hat{n}$ makes a fixed angle $\theta$ with layer normal $\hat{s}$. The ferroelectric polarization $\mathbf{P}$ is in the $x l$ plane and will change direction in this plane when a switching electric field along the $y$ axis is applied. The calculations are simplified in a coordinate system rotated around the $y$ axis. The director is chosen to lie in the $y z^{\prime}$ plane.

In [3], the electrooptic properties were considered for an SSFLC cell with smectic layers perpendicular to the substrates and directors uniformly oriented. If we assume $\theta$ is the smectic $\mathrm{C}^{*}$ tilt angle, $\varphi$ the azimuthal angle, $n_{0}=\left[\varepsilon_{1}\right]^{1 / 2}$ the ordinary refractive index and $\Delta \varepsilon$ the dielectric anisotropy, then the electric field of light transmitted through such an SSFLC slab with thickness $d$ and crossed polarizers is
given by

$$
\begin{align*}
E^{\prime}= & E_{0} \sin [2(\Omega-\gamma)] 2 n_{g}\left[\frac{n_{0} \exp \left(-i n_{0} k_{0} d\right)}{\left(n_{0}-n_{g}\right)^{2}+\left(n_{0}+n_{g}\right)^{2} \exp \left(-2 i n_{0} k_{0} d\right)}\right. \\
& \left.-\frac{n_{0} \exp \left(-i n_{e} k_{0} d\right)}{\left(n_{e}-n_{g}\right)^{2}+\left(n_{e}+n_{g}\right)^{2} \exp \left(-2 i n_{e} k_{0} d\right)}\right] \exp \left(-i n_{g} k_{0} d\right) \tag{1}
\end{align*}
$$

where $n_{e}=\left[\varepsilon_{1}\left(\varepsilon_{1}+\Delta \varepsilon\right) /\left(\varepsilon_{1}+\Delta \varepsilon \sin ^{2} \theta^{\prime}\right)\right]^{1 / 2}, k_{0}=\omega / c, \Omega$ is the angle between polarizer and $z$ axis, $\gamma$ is the angle between $z$ axis and the projection of the director on to the $x z$ plane, $\theta^{\prime}$ is the angle between the director and the $x z$ plane, $n_{g}$ is the refractive index of substrates and $E_{0}$ is the electric field amplitude of incident beam.

If we assume the layer tilt angle is $\delta$, then the transmitted intensity for uniform layer tilted SSFLC cells can be obtained by replacing $\gamma$ and $\theta^{\prime}$ in equation (1) by the quantities defined below:

$$
\begin{align*}
\tan \gamma & =\sin \theta \cos \varphi /(\sin \delta \sin \theta \sin \varphi+\cos \theta \cos \delta)  \tag{2a}\\
\sin \theta^{\prime} & =\cos \delta \sin \theta \sin \varphi-\cos \theta \sin \delta \tag{2b}
\end{align*}
$$

The correction to the equation of motion is a little more complicated. Neglecting the surface interactions [3], the free energy density can be written as

$$
\begin{equation*}
F= \pm P E \cos \delta \cos \varphi-1 /(8 \pi) \Delta \varepsilon E^{2} \cos ^{2} \delta \sin ^{2} \theta\left(\sin \varphi-\sin \varphi_{0}\right)^{2} \tag{3}
\end{equation*}
$$

where $E$ is the applied electric field and $P$ is the spontaneous polarization. The two terms are ferroelectric and dielectric interactions respectively. The ' + ' and ' - ' signs in the first term correspond to the electric field pointing along the positive and negative $y$ axes respectively. $\varphi_{0}$ is defined by $\sin \varphi_{0}=\tan \delta / \tan \theta$ and is found experimentally to be either about $45^{\circ}$ (layer tilt angle smaller than smectic $\mathrm{C}^{*}$ tilt angle) or $90^{\circ}$ (layer tilt angle equal to molecular angle) [5,6]. If $\varphi_{0}$ is less than $90^{\circ}$, then the surface can stabilize two states, $\varphi=\varphi_{0}$ and $\varphi=\pi-\varphi_{0}$, when there is no applied electric field. If $\varphi_{0}=90^{\circ}$, then the surface may only stabilize the state $\varphi=\varphi_{0}$. Minimizing these two free energies with respect to the azimuthal angle $\varphi$ for up and down fields, we find for $\Delta \varepsilon>0$ that the director of the FLC molecules will switch between $\varphi=\pi+\varphi^{\prime}$ and $\varphi=2 \pi-\varphi^{\prime}$, where $\varphi^{\prime}$ satisfies the equation

$$
\begin{equation*}
\sin \varphi^{\prime}=\alpha \cos \varphi^{\prime}\left(\sin \varphi^{\prime}+\sin \varphi_{0}\right) \tag{4}
\end{equation*}
$$

where $\alpha$ is defined as for equation (7). For negative anisotropy ( $\Delta \varepsilon<0$ ), the switching range is $\left[\varphi^{\prime}, \pi-\varphi^{\prime}\right]$ and $\varphi^{\prime}$ is defined by

$$
\begin{equation*}
\sin \varphi^{\prime}=\alpha \cos \varphi^{\prime}\left(\sin \varphi^{\prime}-\sin \varphi_{0}\right) . \tag{5}
\end{equation*}
$$

Adding the torque due to viscosity $\eta$, the motion of the spontaneous polarization $\mathbf{P}$ in an electric field $\mathbf{E}=-E \hat{y}$ is described by the equation

$$
\begin{equation*}
\tau d \varphi / d t=\sin \varphi+\alpha \cos \varphi\left(\sin \varphi-\sin \varphi_{0}\right) \tag{6}
\end{equation*}
$$

Integrating this equation, we get

$$
\begin{equation*}
\frac{t}{\tau}=\int_{\psi}^{\varphi} \frac{\mathrm{d} \varphi}{\sin \varphi+\alpha \cos \varphi\left(\sin \varphi-\sin \varphi_{0}\right)}, \tag{7}
\end{equation*}
$$

where $\tau=\eta /(P E \cos \delta), \alpha=\Delta \varepsilon E^{2} \sin ^{2} \theta \cos \delta /(4 \pi P E)$. The lower limit of integration $\psi$ is $2 \pi-\varphi^{\prime}$ for $\Delta \varepsilon>0$ and $\varphi^{\prime}$ for $\Delta \varepsilon<0$ with $\varphi^{\prime}$ defined in equations (4) and (5) respectively for the two cases. As we can see, the fact that the smectic layer tilts with
respect to substrates changes the time constants by a factor of $1 / \cos \delta$. Also, a term $\alpha \sin \varphi_{0} \cos \varphi$ is added to the torque equation which significantly changes the early stage of the switching process. When the electric field is turned off, the cell will slowly relax back to its nearest surface stabilized state under surface interactions.

## 3. Results and discussion

Using the azimuthal angle $\varphi$ as a parameter, we can calculate the transmission intensity $T(t)$ as a function of time. We assume an electric pulse such that the leading edge switches the cell from spontaneous polarization $\mathbf{P}$ UP to DOWN and the trailing edge switches $\mathbf{P}$ from DOWN to UP, where UP and DOWN states are defined as the states when the angle between polarization $\mathbf{P}$ and the $+y$ axis is $\varphi=\varphi^{\prime}$ and $\varphi=\pi-\varphi^{\prime}$, and wide enough to switch the polarization to saturation. Some typical results of such calculations are shown in figures 3 and 4 respectively for $\Delta \varepsilon>0$ and $\Delta \varepsilon<0$. In the calculations, $\Omega$ is so chosen such that extinction occurs for the starting state. As we can see from equations (4) and (5), the applied electric field changes the switching range $\left[\varphi^{\prime}, \pi-\varphi^{\prime}\right]$. This will change the maximum transmission of the light valve. A typical plot of this maximum transmission versus applied voltage is illustrated in figure 5.

### 3.1. Time constants

We define delay time $t_{\mathrm{d}}$ and rise time $t_{\mathrm{r}}$ as in [3], i.e. $t_{\mathrm{d}}$ is the time taken to obtain 10 per cent of maximum transmission and $t_{\mathrm{r}}$ is the time taken to switch the cell from 10 per cent to 90 per cent of maximum transmission. We also define the fall time


Figure 3. Transmitted light versus time for $\Delta \varepsilon>0$ after the application of a voltage first favouring polarization $\mathbf{P}$ DOWN then UP. In the process from UP to DOWN, transmission reaches 10 per cent of maximum after delay time $t_{\mathrm{d}}$. After a further rise time $t_{\mathrm{r}}$ intensity reaches 90 per cent of maximum. The fall time is defined as the time taken to switching the light valve from 90 per cent to 10 per cent of maximum in the process from DOWN to UP. For this example, $k_{0} d=9.93, n_{g}=1 \cdot 5, \varepsilon_{1}=2 \cdot 19, \Delta \varepsilon=0 \cdot 77, \theta=20^{\circ}$. The four curves correspond to different $\varphi_{0}$ and $\alpha$ : (a) $\varphi_{0}=45^{\circ}, \alpha=0 \cdot 1$; (b) $\varphi_{0}=45^{\circ}$, $\alpha=0 \cdot 2 ;(c) \varphi_{0}=90^{\circ}, \alpha=0 \cdot 1 ;(d) \varphi_{0}=90, \alpha=0 \cdot 2$.


Figure 4. Transmitted light versus time for $\Delta \varepsilon<0 . \varepsilon_{1}=2 \cdot 5, \Delta \varepsilon=-1$. Other parameters are the same as for figure 3 except signs changed for $\alpha$ s.


Figure 5. Maximum transmission versus applied electric field. The two lower curves are for $\Delta \varepsilon>0$ using parameters for figure 3 with (a) $\varphi_{0}=45^{\circ}$ and (b) $\varphi_{0}=90^{\circ}$, and the two upper curves for $\Delta \varepsilon<0$ using parameters for figure 4 with (c) $\varphi_{0}=45^{\circ}$ and (d) $\varphi_{0}=90^{\circ}$.
$t_{\mathrm{f}}$ as the time taken to switch the cell from 90 per cent to 10 per cent of maximum transmission by the trailing edge of the switching pulse. Some typical results of these time constants are plotted in figures 6 and 7 for $\Delta \varepsilon>0$ and $\Delta \varepsilon<0$ respectively. As seen in the graphs, the rise time and fall time in either case follows close to a $1 / E$ dependence on the field strength, agreeing with experimental results $[1,8]$. The delay time for $\Delta \varepsilon<0$ never diverges, as would happen with smectic layers perpendicular


Figure 6. Rise time, $t_{\mathrm{r}}$, delay time $t_{\mathrm{d}}$ and fall time $t_{\mathrm{f}}$ versus applied field $E$ for $\Delta \varepsilon>0$. Dashed lines show $t \propto 1 / E$ and $t \propto 1 / E^{2}$ dependence. Parameters are the same as in figure 3 with (a) $\varphi_{0}=45^{\circ}$ and (b) $\varphi_{0}=90^{\circ}$.
to the substrates. This is because the dielectric energy is minimum at $\varphi=\varphi_{0}$, so when a switching field is applied, both dielectric torque and ferroelectric torque are pulling the spontaneous polarization away from its initial state $\varphi=\varphi^{\prime}$. At high field, the maximum transmitted light intensity for $\Delta \varepsilon>0$ drops dramatically, and the time constants $t_{\mathrm{d}}, t_{\mathrm{r}}$ and $t_{\mathrm{f}}$ all follow a nearly $1 / E^{2}$ law in this case due to the dominance of the dielectric effect.


Figure 7. Rise time $t_{\mathrm{r}}$, delay time $t_{\mathrm{d}}$ and fall time $t_{\mathrm{f}}$ versus applied field $E$ for $\Delta \varepsilon<0$. Parameters are the same as in figure 4 with (a) $\varphi_{0}=45^{\circ}$ and (b) $\varphi_{0}=90^{\circ}$.

### 3.2. Asymmetry characteristics of the switching process

As seen in figures 3-7, the optical response in general is not symmetric. This is especially true for $\Delta \varepsilon>0$, where the difference between rise time $t_{\mathrm{r}}$ and fall time $t_{\mathrm{f}}$ can be very large.

Assuming at $\varphi_{1}$ the light valve transmits 10 per cent of the maximum intensity and at $\varphi_{9} 90$ per cent, then the rise time is the time taken to change $\varphi$ from $\varphi_{1}$ to $\varphi_{9}$. The fall time will be the time taken to change $\varphi$ from $\pi-\varphi_{9}$ to $\pi-\varphi_{l}$ under the same conditions. These two times will be different unless $\varphi_{1}=\pi-\varphi_{9}$ or $\varphi$ is a linear function of time.



Figure 9. Chevron structure of SSFLC cells. The top and bottom of the smectic layers tilt in different ways but make the same angle with the substrates. The polarization $\mathbf{P}$ in this case is splayed for either UP or DOWN state. When a switching field is applied, polarization $\mathbf{P}$ in the halves of the cell with $+\delta$ and $-\delta$ layer tilts rotates in opposite ways, causing a kink defect at the intersection.

For comparison with the discussion in [7], we calculated the transmission response $T(t)$ when the polarizer was set to make an angle of $22 \cdot 5^{\circ}$ with the $z$ axis (or the FLC layer normal if a straight smectic layer arrangement is assumed) and the time constants were defined by the time from the up or down edge of the applied switching pulse to the 50 per cent change of transmission. Some results are shown in figure 8 as a function of applied field for $\Delta \varepsilon>0$. For $\Delta \varepsilon<0$, these two times are almost the same.

So far we have dealt with the case where layers are tilted in one direction. However, very often the smectic layers are found to have a so-called chevron structure [5], the top part of the layers tilting one way $(+\delta)$ and the bottom part of the layers tilting the other way $(-\delta)$, as illustrated in figure 9 . The switching process in the bottom part of the cell $(-\delta)$ can be treated by replacing $\delta$ by $-\delta$ in equations above for the $+\delta$ part of the cell. We found that, when an electric field is applied, the spontaneous polarization UP state is $\varphi=\varphi^{\prime}$ and the DOWN state is $\pi-\varphi^{\prime}$ for $-\delta$ while the corresponding states for the $+\delta$ part of the cell are $\varphi=2 \pi-\varphi^{\prime}$ and $\pi+\varphi^{\prime}$. When a switching step voltage is applied, the azimuthal angle changes from $2 \pi-\varphi^{\prime}$ to $\pi+\varphi^{\prime}$ (rotating clockwise) in the top part $(+\delta)$ and from $\varphi^{\prime}$ to $\pi-\varphi^{\prime}$ (rotating counterclockwise) in the bottom part $(-\delta)$ of the cell. At the intersection we have a $2 \pi$ wall [9] whose thickness is the electric correlation length $\xi=\left(K_{1} / P E\right)^{1 / 2}$, where $K_{1}$ is the splay elastic constant. In all but the direction of rotation, these two parts of the cell are identical, i.e. the transmission-time relation is the same for two parts when a switching field is applied.

In the intermediate-high field regime, chevron $\operatorname{kink} \xi$ and surface layers are all very small compared to both the light wavelength and the sample thickness [3], and contributions to optical transmission from these parts of the cell are negligible. The calculations in this paper then should be a good approximation.

Figure 8. Rise time $t_{+}$, defined as the time from the leading edge of the switching voltage to 50 per cent change of the transmission, and fall time $t_{-}$, defined as the time from the trailing edge of the switching voltage to 50 per cent of transmission, versus applied electric field for $\Delta \varepsilon>0$. The polarizer was set to make an angle $22 \cdot 5^{\circ}$ with the $z$ axis. Parameters are the same as in figure 3 with (a) $\varphi_{0}=45^{\circ}$ and (b) $\varphi_{0}=90^{\circ}$. (c) shows a typical electrooptic response with an applied field in this case.

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